Anomalous crossover behavior at finite temperature

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We introduce a stochastic growth model where the growth is controlled by a temperaturelike parameter T. The model shows various types of dynamical behavior as T changes from 0 to ∞. For $T=0$ the growth process belongs to the quenched Kardar-Parisi-Zhang (KPZ) universality class, whereas it belongs to the Edwards-Wilkinson (EW) universality class for $T=\infty$. In the intermediate range $0<T<\infty$, the model shows an anomalous crossover behavior from the quenched KPZ to the thermal KPZ class. The KPZ nonlinearity is generated by an anisotropic effect of the quenched noise which exists only for $T<\infty$ in our model. We also study crossovers between different types of scaling behavior of the interface width for various T's.

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Recently the dynamics of driven interfaces in random media has attracted much attention [1–3]. The driven motion of an interface in a random medium is governed by an interplay between the quenched disorder of the random medium and the external driving force acting on the interface. The interface is pinned when the driving force F is smaller than the pinning strength induced by the quenched disorder. On the other hand, the interface moves with a constant velocity when F is larger than the pinning strength. Hence, there is a threshold of the driving force F_c above which the interface moves with a constant velocity; the velocity is zero for $F<F_c$ and it increases for $F>F_c$. This phenomenon is referred to as a pinning-depinning transition.

Near the depinning threshold, the dynamics of a driven interface in a random medium shows a nontrivial scaling behavior of the interface width,

$$W(L,t)\equiv\left\langle \frac{1}{L^d}\sum_{\mathbf{x}}[h(\mathbf{x},t)-\overline{h}(t)]^2\right\rangle^{1/2}, \quad (1)$$

where $h(\mathbf{x},t)$ is the interface height at site $\mathbf{x}$ on the substrate at time t. Here $\overline{h}(t)$, L, and d denote the mean height at time t, the system size, and the substrate dimension, respectively. The symbol $\langle\cdots\rangle$ stands for the statistical average over many realizations of randomness. The interface width shows a scaling behavior $W\sim L^\alpha f(t/L^z)$, where the scaling function $f(x)$ approaches a constant for $x\gg 1$ and scales as $f(x)\sim x^\beta$ for $x\ll 1$ with $z=\alpha/\beta$ [4]. The exponents α, β, and z are called the roughness, the growth, and the dynamic exponent, respectively.

The dynamics of a driven interface in a random medium can be explained by a Langevin-type continuum equation. The well-known nonlinear equation describing the motion of a driven interface in random media is the quenched Kardar-Parisi-Zhang (QKPZ) equation [5,6]

$$\frac{\partial h(\mathbf{x},t)}{\partial t}=\nu\nabla^2 h(\mathbf{x},t)+\frac{\lambda}{2}[\nabla h(\mathbf{x},t)]^2+F+\eta(\mathbf{x},h), \quad (2)$$

where F is a driving force. The quenched noise satisfies $\langle\eta(\mathbf{x},h)\rangle=0$ and $\langle\eta(\mathbf{x},h)\eta(\mathbf{x}',h')\rangle=\delta^d(\mathbf{x}-\mathbf{x}')\Delta(h-h')$. Here $\Delta(h-h')$ is assumed to be a monotonically decreasing function of $h-h'$ for $h-h'>0$ and decays exponentially to zero over a finite distance a. The quenched noise term describes a random force which is induced by the quenched disorder. Many numerical works have been carried out to describe and understand the motion of driven interfaces described by the QKPZ equation. The roughness exponent $\alpha\simeq 0.63$ in one dimension was obtained from numerical and analytical studies [7,8]. For $\lambda=0$, however, Eq. (2) belongs to another universality class called the quenched Edwards-Wilkinson (QEW) class. Analytical [9] and numerical [10–13] studies of the QEW class yield a roughness exponent $\alpha=1\sim 1.25$ in one dimension.

The physical properties of a growing interface in a homogeneous medium are different from those in random media. In homogeneous media, the dynamics of a growing interface is influenced by a white noise, which is an uncorrelated random noise with strength of D, instead of a quenched noise. [Note that Eq. (2) becomes the KPZ equation [14] when the quenched noise is replaced by a white noise.] Moreover, when λ is also 0, Eq. (2) turns into a simple linear equation called the Edwards-Wilkinson (EW) equation [15]. By solving the EW equation directly, one can easily obtain the values of the roughness and dynamic exponents. The values of the roughness and dynamic exponents are $\alpha=(2-d)/2$ and $\beta=(2-d)/4$, respectively. A well-known model belonging to the EW universality class is the Family model [16].

Several years ago, Vergeles [17] studied the competition effect between quenched and thermal noise by introducing a temperaturelike parameter T in the Sneppen model [7] belonging to the QKPZ universality class. In this model, a growth process occurs at site x with a probability proportional to $\exp[-q(x)/T]$ for $0<T<\infty$, where $q(x)$ represents impurities in random media. The model corresponds to the Sneppen model [7] for $T=0$ and the restricted solid-on-solid (RSOS) model [18] belonging to the KPZ universality class if $T=\infty$. For $T=\infty$, the growth process occurs with equal probability at any site so that the growth mechanism is the same as that of a growing interface in a homogeneous medium. For finite T, the growing probability at a site having small $q(x)$ becomes larger compared to sites having a large value of $q(x)$. Therefore, the effect of quenched noise in

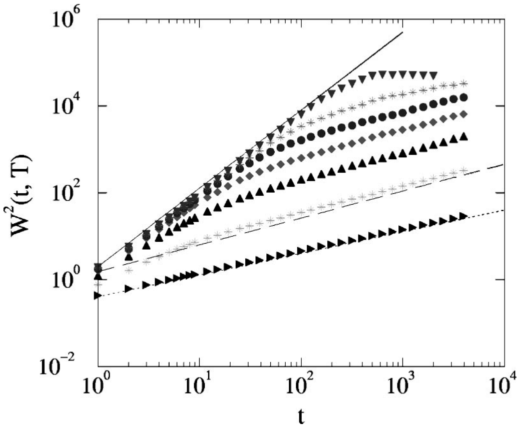

FIG. 1. The plots of width $W^2(t,T)$ vs time t are shown for $1/T = 0, 2, 4, 6, 8, 10$, and ∞ from the bottom to the top with system size $L=10\,000$. The slopes of the solid, long-dashed, and dotted lines are 1.8, 0.62, and 0.5, respectively.

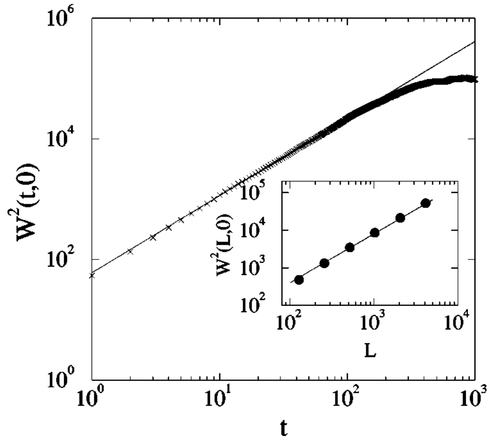

FIG. 2. The growth exponent β_s starting from the saturated interface for $T=0$. The straight guide line represents $\beta_s \approx 0.64$. In the inset, we obtained the roughness exponent $\alpha \approx 0.65$.

random media can be induced by tuning the temperaturelike parameter T. Vergeles, however, found that although thermal noise and quenched noise coexist in the model for $0<T<\infty$, the model belongs to the KPZ universality class. The reason is that the effect of thermal noise is more dominant in interface growth than the effect of the quenched noise when the two noises coexist.

In this paper, we introduce a simple stochastic growth model where the growth rate is controlled by a temperature-like parameter T. The growth rule of the model is the same as the one in the original Family model [16] which is known to belong to the EW universality class when $T=\infty$. Interestingly, for $T=0$ we find that our model does *not* belong to the QEW universality class; instead, it belongs to the QKPZ universality class. The KPZ nonlinearity observed in our model originates from an anisotropic effect of the quenched noise. For $0<T<\infty$, our model shows a crossover from the QKPZ to the KPZ universality class. This means that thermal noise is more dominant than quenched noise in our model. The same applies to Vergeles' model, but in the present case the KPZ nonlinearity generated by the anisotropy of the quenched noise persists without disappearing for $0<T<\infty$.

The stochastic rule of our model is defined as follows: Before the simulation starts, we preassign random numbers $q(x)$, representing impurities in the random medium, to all perimeter sites of the initially flat interface. During the temporal evolution, a site is selected with a probability proportional to $\exp[-q(x)/T]$, where $0<T<\infty$, and a particle is deposited on that site. If the heights of the nearest-neighbor sites are lower than that of the selected site, the deposited particle is allowed to diffuse to the nearest-neighbor site with the smaller height. Then the random number at the newly occupied site is updated. As already mentioned, the model corresponds to the Family model in the EW universality class at $T=\infty$.

Our simulations were carried out starting from a flat initial condition with periodic boundary conditions in one dimension. Numerical data were averaged over 100 independent runs. Figure 1 shows the plot of the surface width $W^2(t)$ as a function of the time t using the system size $L=10\,000$ for $1/T=0, 2, 4, 6, 8, 10$, and ∞. The straight guide lines represent $\beta=0.25$ for the dotted line, $\beta=0.31$ for the long-dashed line, and $\beta=0.9$ for the solid line. For $T=\infty$, $\beta \approx 0.25$ is the expected value from the Family model, i.e., the EW universality class. On the other hand, when $T=0$, we obtained $\beta \approx 0.9$. For finite temperatures, the surface width $W(t,T)$ shows a crossover behavior from $t^{0.9}$ to $t^{0.31}$. The value of $\beta \approx 0.31$ is close to the one expected from the KPZ universality class.

To confirm the universality class of our model in the case $T=0$, we considered another growth exponent β_s when $T=0$. Here β_s was measured on the initially saturated interface instead of the flat interface. Figure 2 shows the plot of the interface width $W^2(t,0)$ versus the time t. The straight guide line represents $\beta_s \approx 0.64$. The value of the growth exponent β_s obtained from the saturated interface is generally smaller than that of the growth exponent β from a flat interface in growth models of driven interface in random media. The growth exponent obtained from the saturated interface is well known to be the correct growth exponent to classify the universality class. Therefore, the obtained value of the growth exponent indicates that our model belongs to the QKPZ universality class. We also measured the roughness exponent α. As shown in the inset of Fig. 2, the obtained roughness exponent is $\alpha \approx 0.65$. This value is also almost the same as 0.63 expected from the QKPZ class. Thus we conclude that our model belongs to the QKPZ universality class when $T=0$.

In general, the KPZ nonlinearity $[\lambda(\nabla h)^2/2]$ is known to have a kinematic origin when the velocity v of the growing interface is nonzero and proportional to λ [14]. In the case of a driven interface near the depinning threshold, the velocity of a driven interface is zero or almost zero. However, the critical exponents obtained from experiments and discrete models for a driven interface in random media indicate the existence of a KPZ nonlinearity. Several years ago, Tang, Kardar, and Dar (TKD) [6] argued that the anisotropy in random media can be a possible source of the KPZ nonlinearity for driven interfaces in random media. They argued that the hallmark of the anisotropic depinning is the depen-

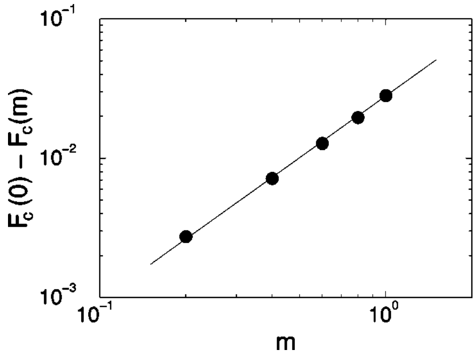

FIG. 3. The plot of $F_c(0)-F_c(m)$ vs m is shown for the system size $L=1024$. The threshold force linearly depends on the tilt of the interface.

dence of the depinning threshold $F_c(m)$ on the slope of the tilted substrate m,

$$F_c(m)-F_c(0)\propto -|m|^{1/\nu(1-\alpha)}, \tag{3}$$

where the correlation length parallel to the interface, ξ, scales as $\xi \sim (F-F_c)^{\nu}$ and ν is called the correlation length exponent.

In order to check whether there is anisotropic depinning in our model at $T=0$, it is necessary to know the exact value of $F_c(m)$ in our model. By measuring the distribution of random numbers and the distribution of the minimum random numbers in the critical state [19], we obtained the threshold force $F_c(m)$ against the tilt of the substrate m. We plotted $F_c(m)$ versus the slope m in a double logarithmic scale in Fig. 3. We found that the data appear as a straight line in the log-log plot. The slope of the line obtained from a least square fit is 1.47 ± 0.02. This value is in a fairly good agreement with the value expected from $1/\nu(1-\alpha)$, where $\nu=1.7$ and $\alpha=0.63$ are the exponents of the QKPZ universality class. This means that for $T=0$ anisotropy effects play a role in our model and that they generate the KPZ nonlinearity. For finite T, however, quenched and thermal noise coexist in our model. The effect of thermal noise is more dominant than that of quenched noise in the regime where the quenched and thermal noise coexist. Interestingly, our model does not belong to the EW class but belongs the KPZ universality class for finite T. The KPZ nonlinearity, which originates from anisotropic effect of the quenched noise, does not disappear and persists even in the presence of thermal noise in our model. As shown in Fig. 1, there is indeed a crossover from the QKPZ to the KPZ class in the model for $0<T<\infty$. We considered the crossover scaling behavior of the interface width for various T, as will be described in the following paragraphs.

The scaling behavior of interface width $W^2(t,T)$ changes with the variation of a temperaturelike parameter T. We considered the crossover scaling of the interface width for various T's. Our analysis shows that these results are well described by

$$W^2(t,T) \;=\; A(T)\; t^{2\beta_{kpz}}\; F(t\;\exp(-g/T)) \tag{4}$$

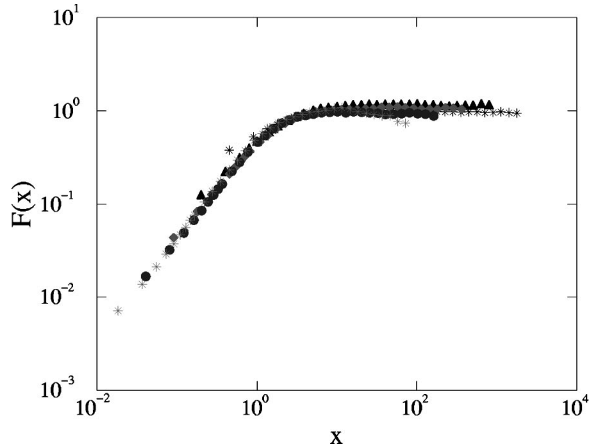

FIG. 4. Data collapse of $W^2(t,T)$ according to Eq. (4) for $1/T=2$, 4, 6, 8, and 10, using $g\approx -0.4$ and $A(T)\approx e^{1.3\,\ln(1/T)^{1.74}}$.

where $F(x)\rightarrow 1$ for $x\rightarrow\infty$ and $F(x)\rightarrow x^{\beta_{qkpz}-\beta_{kpz}}$ for $x\rightarrow 0$. Here β_{kpz} and β_{qkpz} are the growth exponents for the KPZ and QKPZ universality classes, respectively. Figure 4 shows the collapse of the $F(x)$ versus x. The data are well collapsed onto a single curve with $A(T)\approx e^{1.3\,\ln(1/T)^{1.74}}$ and $g\approx -0.4$.

We also measured the roughness exponent. Figure 5(a) shows the plot of the interface width $W^2(L,T)$ versus L for $1/T=$ 2, 4, 6, 8, and 10. The dotted line represents the estimate $\alpha\simeq 0.5$ and the solid line represents the estimate $\alpha\simeq 0.65$. Thus we find that for $T=0$ the model belongs to the QKPZ universality class, while for $0<T<\infty$, the model shows the crossover behavior from the QKPZ to the KPZ class. The scaling behavior of interface width $W^2(L,T)$ satisfies the following scaling form:

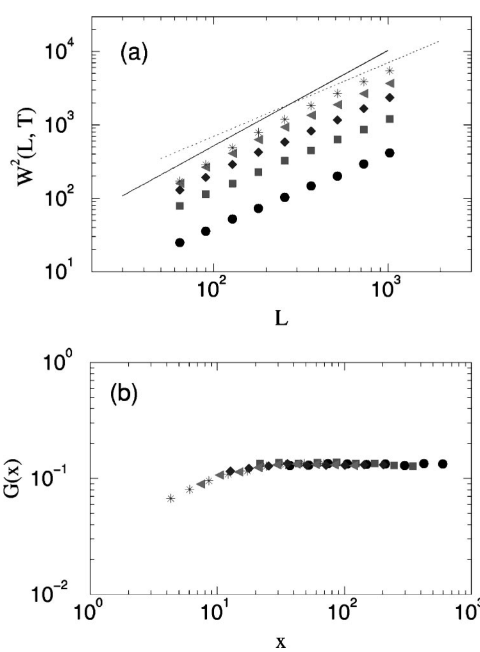

FIG. 5. (a) The plot of width $W^2(L,T)$ vs L is shown for $1/T=2$, 4, 6, 8, and 10 from the bottom to the top. The solid line represents $\alpha\approx 0.65$ and the dotted line shows $\alpha\approx 0.5$. (b) Data collapse of $W^2(L,T)$ according to Eq. (5).

$$W^2(L,T) \sim B(T)\ L^{2\alpha_{kpz}}\ G(L\ \exp(-g/T)), \quad (5)$$

where $G(x)\rightarrow 1$ for $x\rightarrow\infty$ and $G(x)\rightarrow x^{\alpha_{qkpz}-\alpha_{kpz}}$ for $x\rightarrow 0$. Here α_{kpz} and α_{qkpz} are the roughness exponents of the KPZ and QKPZ universality classes, respectively. Figure 5(b) shows the collapse of $G(x)$ versus x. The data are well collapsed onto a single curve with $B(T)\approx T^{-1.6}$ and $g \approx -0.27$.

In summary, we have studied the generalized Family model by introducing a temperaturelike parameter T. For $T=\infty$, the growth rule of our model is the same as that of the Family model in the EW universality class, whereas our model belongs to the QKPZ universality class for $T=0$. By tuning the parameter T (and therewith the intensity of the quenched noise) we find that the interface width shows a crossover from the QKPZ to the KPZ universality class. Here the crossover time and the crossover length depend on the parameter T exponentially. We also determined the scaling functions for the interface width $W^2(t,T)$ and $W^2(L,T)$. This crossover behavior is anomalous and originates from the competition of quenched and thermal noise. When $T=\infty$, our model belongs to the EW class, where the KPZ nonlinearity is absent. For finite T the KPZ nonlinearity is also generated by the anisotropic effect of the quenched disorder. Apparently, the effect of thermal noise is more dominant in the interface growth than that of the quenched noise when two noises coexist. However, our model belongs to the KPZ class rather than the EW class for $0<T<\infty$. This means that the KPZ nonlinearity by anisotropic effect of the quenched noise persists without disappearing in the presence of the thermal noise.

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[1] M. A. Rubio, C. A. Edwards, A. Dougherty, and J. P. Gollub, Phys. Rev. Lett. **63**, 1685 (1989); D. Kessler, H. Levine, and Y. Tu, Phys. Rev. A **43**, 4551 (1991).

[2] O. Narayan and D. S. Fisher, Phys. Rev. B **46**, 11 520 (1992).

[3] D. Ertas and M. Kardar, Phys. Rev. Lett. **73**, 1703 (1994); Phys. Rev. B **53**, 3520 (1996); B. Kahng, K. Park, and J. Park, Phys. Rev. E **57**, 3814 (1998).

[4] *Dynamics of Fractal Surfaces*, edited by F. Family and T. Vicsek (World Scientific, Singapore, 1991); T. Vicsek, *Fractal Growth Phenomena*, (World Scientific, Singapore, 1992); A. L. Baráabasi and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, Cambridge, England, 1995).

[5] L. A. N. Amaral, A.-L. Barabási, and H. E. Stanley, Phys. Rev. Lett. **73**, 62 (1994).

[6] L.-H. Tang, M. Kardar, and D. Dhar, Phys. Rev. Lett. **74**, 920 (1995).

[7] K. Sneppen, Phys. Rev. Lett. **69**, 3539 (1992).

[8] H. Leschhorn, Phys. Rev. E **54**, 1313 (1996).

[9] T. Nattermann, S. Stepanow, L.-H. Tang, and H. Leshhhorn, J. Phys. II **2**, 1483 (1992); O. Narayan and D. S. Fisher, Phys. Rev. B **48**, 7030 (1993).

[10] M. Dong, M. C. Marchetti, A. A. Middleton, and V. Vinokur, Phys. Rev. Lett. **70**, 662 (1993).

[11] H. Leshhorn, Ph. D. thesis, Ruhr University, Bochum, 1994.

[12] S. Roux and A. Hansen, J. Phys. I **4**, 515 (1994); H. J. Jensen, J. Phys. A **28**, 1861 (1995); H. A. Makse and L. A. N. Amaral, Europhys. Lett. **31**, 379 (1995); L. A. N. Amaral, A.-L. Barabási, H. A. Makse, and H. E. Stanley, Phys. Rev. E **52**, 4087 (1995).

[13] K. Park and In-mook Kim, Phys. Rev. E **59**, 5150 (1999).

[14] M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986).

[15] S. F. Edwards and D. R. Wilkinson, Proc. R. Soc. London, Ser. A **381**, 17 (1982).

[16] F. Family, J. Phys. A **19**, L441 (1986).

[17] M. Vergeles, Phys. Rev. Lett. **75**, 1969 (1995).

[18] J. M. Kim and J. M. Kosterlitz, Phys. Rev. Lett. **62**, 2289 (1989).

[19] P. Bak and K. Sneppen, Phys. Rev. Lett. **71**, 4083 (1993).